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Running U_{e3} and BR($\mu \to e + \gamma$) in SUSY-GUTs

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Abstract: In supersymmetric seesaw models based on SUSY-GUTs, it could happen that the neutrino PMNS mixing angles are related to the lepton flavour violating decay rates. In particular SO(10) frameworks, the smallest mixing angle would get directly correlated with the $\mu \to e + \gamma$ decay amplitude. Here we study this correlation in detail considering U_{e3} as a free parameter between 0 and U_{e3}(CHOOZ). Large radiative corrections to U_{e3} present in these models, typically of the order $\Delta U_{e3} \sim 10^{-3}$ (peculiar to hierarchial neutrinos), can play a major role in enhancing the Br($\mu \to e + \gamma$), especially when $U_{e3} \lesssim 10^{-3}$. For large tanβ, even such small enhancements are sufficient to bring the associated Br($\mu \to e + \gamma$) into realm of MEG experiment as long as SUSY spectrum lies within the range probed by LHC. On the other hand, for some (negative) values of U_{e3}, suppressions can occur in the branching ratio, due to cancellations among different contributions. From a top-down perspective such low values of U_{e3} at the weak scale might require some partial/full cancellations between the high scale parameters of the model and the radiative corrections unless U_{e3} is purely of radiative origin at the high scale. We further emphasize that in Grand Unified theories there exist additional LFV effects related to the running above the GUT scale, that are also independent on the low energy value of U_{e3}. These new contributions can become competitive and even dominant in some regions of the parameter space.

Keywords: Supersymmetry Phenomenology, Beyond Standard Model, GUT.

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Contents

1. Introduction and conclusions

It is well know that, after the discovery that neutrinos are massive, the detection of supersymmetry (SUSY) induced lepton flavour violation (LFV) processes has become a very interesting possibility. This is specifically true in the presence of a seesaw like mechanism being operative at the high scale leading to small non-zero neutrino masses as well as large mixing in the neutrino sector. The potential of this admixture of SUSY and seesaw [1] and its implications to lepton flavour violation has been studied in several papers in the last few years [2], especially in the light of upcoming experiments like MEG [3].

Thanks to the RG evolution (in the presence of heavy right-handed neutrinos) from the high scale to the low energies where experiments are conducted, SUSY seesaw leads to potentially sizable mixing effects in the slepton mass matrices, which give rise to flavour violating charged leptonic decays through loop-induced processes. These flavour violating effects are strongly dependent on the neutrino Yukawa matrix, Y_ν whose entries are generically unknown. Fortunately, the seesaw mechanism fits nicely within the larger picture of SUSY Grand Unification (GUTs) especially in models based on SO(10) gauge groups. A quite general feature of SUSY SO(10) is the relation among the Y_ν and up-quark Yukawa matrix Y_u eigenvalues, which ensures that at least one of the neutrino Yukawa is as large as the top Yukawa y_t [4].

In previous works [4, 5], we considered two benchmark cases in which the mixing angles in Y_ν were either minimal (CKM-like) or maximal (PMNS-like). In those works, we have studied the implications of these two benchmark scenarios for the (indirect) discovery potential of SUSY in the various up-coming experiments like MEG, PRISM/PRIME, and super-B factories. We have found that experimental sensitivity of these experiments can be quite complementary with the direct discovery machine, Large Hadron Collider (LHC) at CERN and in some cases, could far outreach the sensitivity of LHC itself. These latter cases are the ones which have large (maximal) mixing in the neutrino Yukawa couplings going by the name, “PMNS-case”. A particular feature of this case is that some of the LFV processes, such as $\mu \to e\,\gamma$, turn out to be dependent on the so far unknown U_{e3}

entry of the mixing matrix $U_{\rm PMNS}$. The aim of the present paper is to study the correlation between U_{e3} and the LFV decay, $\mu \to e\,\gamma$, in a SO(10) scenario with mSUGRA boundary conditions. To this extent, we will treat the low energy value of U_{e3} as an independent parameter varying between $0 \lesssim U_{e3} \lesssim U_{e3}$(CHOOZ) [6].

In studying the variation of the BR($\mu \to e+\gamma$) with respect to the unknown neutrino mixing angle U_{e3} a crucial factor turns out to be the RG effects on the neutrino mass matrices themselves from the weak scale to the scale of right-handed neutrinos and further up to the GUT scale (in terms of the seesaw parameters). The main point is that even small U_{e3} can be sufficient to generate large corrections to the BR, thanks to running effects of U_{e3} itself. In fact, the crucial parameter in computing BR($\mu \to e+\gamma$) turns out to be the high-energy value of U_{e3}, instead the low-energy one, which can be measured by neutrino oscillations experiments. Even small values of the U_{e3} generated at the high scale $\sim \mathcal{O}(10^{-3} - 10^{-4})$ could significantly modify the RG generated flavour off-diagonal entries in the slepton mass matrices and thus enhance the branching ratios. Perhaps, the most striking aspect of this appears in the predictions of the branching ratios for $\mu \to e+\gamma$ at the MEG experiment in the SUSY-GUT parameter space being probed by LHC. In fact, including these effects would enhance the predictions for the branching ratios by an order to a couple of orders of magnitude, thus predicting a positive signature for $\mu \to e+\gamma$, at least in the large $\tan\beta$ regime. This particular aspect has been already pointed in passing in our previous work [5]. More recently, the correlation between U_{e3} and other low energy observables in a purely bottom-up approach has been the subject of a thorough analysis of Antusch et al. [7]. We'll comment more about the complementarity of their analysis with our present study. Finally, we note that we resort to purely phenomenological approach without worrying about aspects of flavour model building and origins of U_{e3} and other mixing angles in this work. Such an interesting and important analysis will be treated elsewhere.

In the present work, we study the implications of *running* U_{e3} within the context of SUSY-GUTs. An important point to emphasize is that in SUSY-GUTs there exist other LFV contributions, which rely on the running from the superlarge scale of supergravity breaking down to the GUT scale and are independent of U_{e3}. This paper intends to study the interplay of the two above mentioned sources of LFV for the observability of $\mu \to e+\gamma$. The two sources will be discussed and compared in section II and III. The main conclusions will be drawn in section IV.

2. Running U_{e3} and $(\Delta_{LL})_{12}$

The correlation between U_{e3} and $\mu \to e+\gamma$ within the context of SUSY seesaw has been pointed out long ago [10] and further reviewed by various authors in the recent times [3, 7]. In SUSY seesaw models, the crucial point is the strong dependence of LFV effects on the unknown Y_ν matrix. Such uncertainty is due to the fact that the high-energy parameters entering the seesaw mechanism (Y_ν, M_R) cannot be obtained in terms of the low-energy neutrino masses and mixings (m_{ν_k}, U_{PMNS}), simply because the number of the high-energy parameters is larger. Even within a general SUSY SO(10) framework where the eigenvalues

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of Y_ν are related to the up-quark Yukawas, it is necessary to make assumptions about the mixing structure of Y_ν. An interesting possibility is the case in which the mixing angles result to be PMNS-like. This is what we called PMNS (maximal) mixing case [4, 5]. Here the 'left'-mixing present in the neutrino Yukawa matrix follows the neutrino mixing matrix and is given as:

$$Y_\nu = U_{\mathrm{PMNS}} Y_u^{\mathrm{diag}}, \tag{2.1}$$

The boundary condition given in eq. (2.1) can be, for instance, achieved starting from the SO(10) superpotential [8]:

$$W_{SO(10)} = (Y_u)_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_u + (Y_d)_{ii} \mathbf{16}_i \mathbf{16}_i \frac{\langle \mathbf{45} \rangle}{M_{\mathrm{Planck}}} \mathbf{10}_d + (Y_R)_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{126} \tag{2.2}$$

where **16** is the SO(10) matter representation (i, j are flavour indices), $\mathbf{10}_u$, $\mathbf{10}_d$, **45** and **126** are Higgs representations and Y_u, Y_d and Y_R are Yukawa couplings. The term with **126** uniquely gives rise to right-handed neutrinos masses.

In the case eq. (2.1) holds, the correlation between U_{e3} and $\mu \to e + \gamma$ arises naturally.

Note that the relation of eq. (2.1) is valid at the high scale and thus it is important to evaluate all the entries of the RH-side of this equation at the high scale from their known values at the weak scale. Generically, given that neutrino running effects are small for hierarchical neutrino spectra, the running effects on the PMNS matrix appearing above are neglected. While this is true for the other two of the angles in the PMNS matrix, any small correction to U_{e3} can have significant impact on the value of the radiatively generated $(\Delta_{LL})_{12}$ entry in the slepton mass matrix.

As is well known, the form of the Yukawa matrix feeds into the flavour violating LL entries of the slepton mass matrix through the well known RG effects. At the leading log level, this expression is given by:

$$(\Delta_{LL})_{i \neq j} = -\frac{3m_0^2 + A_0^2}{16\pi^2} \sum_k Y_{\nu\, ik} Y^\dagger_{\nu\, kj} \ln\left(\frac{M_X^2}{M_{R_k}^2}\right) \tag{2.3}$$

where m_0 and A_0 are the common soft scalar mass and trilinear coupling, M_{R_k} the right-handed neutrinos masses and M_X the energy scale at which the SUSY breaking terms appear (coincident, in our framework, with the SO(10) breaking scale). We have used the notation for the slepton mass matrices as

$$\mathcal{M}_{\tilde{l}}^2 = \begin{pmatrix} \Delta_{LL} & \Delta_{LR} \\ \Delta_{RL} & \Delta_{RR} \end{pmatrix}, \tag{2.4}$$

where all the entries on the r.h.s. are matrices in flavour space. Expanding eq. (2.3) using the neutrino Yukawas of eq. (2.1) we have for the 12 or equivalently $e\mu$ entry:

$$(\Delta_{LL})_{12} = -\frac{3m_0^2 + A_0^2}{16\pi^2} \left(y_t^2\, U_{e3} U_{\mu 3}^* \ln\left(\frac{M_X^2}{M_{R_3}^2}\right) + y_c^2\, U_{e2} U_{\mu 2}^* \ln\left(\frac{M_X^2}{M_{R_2}^2}\right) + y_u^2\, U_{e1} U_{\mu 1}^* \ln\left(\frac{M_X^2}{M_{R_1}^2}\right) \right) \tag{2.5}$$

The U_{e3} dependence of $(\Delta_{LL})_{12}$ is clear from the above equation. Importantly, as we see from above, U_{e3} couples with the dominant contribution $\propto y_t^2 \sim \mathcal{O}(1)$. As a consequence, a vanishing value of U_{e3} would strongly suppress the flavour violating mass insertion (MI) by a factor $y_c^2/y_t^2 \sim 10^{-4}$. For small values of U_{e3}, the term with top quark contribution would begin to dominate, once U_{e3} crosses the limit value:

$$|U_{e3}^{\rm lim}| \approx \frac{y_c^2}{y_t^2} \frac{|U_{e2}| \cdot |U_{\mu 2}|}{|U_{\mu 3}|} \frac{\ln M_X - \ln M_{R_2}}{\ln M_X - \ln M_{R_3}} \sim \mathcal{O}(10^{-5}), \tag{2.6}$$

where we have taken M_X to be of the order 10^{17} GeV. Here and throughout the paper, the best fit values [9] for the neutrino oscillations parameters were used ($\Delta m^2_{\rm sol} = 7.9 \cdot 10^{-5}\,{\rm eV}^2$, $\Delta m^2_{\rm atm} = 2.6 \cdot 10^{-3}\,{\rm eV}^2$, $\sin^2\theta_{12} = 0.3$, $\sin^2\theta_{23} = 0.5$), apart from U_{e3} that is considered as a free parameter, as mentioned above. Further, for illustrative purposes, here and later, we will be considering the limit where the low scale value of U_{e3} $\to 0$. The right-handed neutrinos masses M_{R_k} were obtained by solving the seesaw equation (this is possible without uncertainties thanks to the ansatz on the form of Y_ν). The values we found for the M_R eigenvalues are:

$$M_{R_1} = 3.4 \cdot 10^6 {\rm GeV}; \quad M_{R_2} = 2.9 \cdot 10^{10} {\rm GeV}; \quad M_{R_3} = 1.7 \cdot 10^{14} {\rm GeV} \tag{2.7}$$

The above analysis assumes a leading log approximation, where the r.h.s. of the eqs. (2.5), (2.6) are typically assumed to be constant and taken to be their weak scale values, whereas the full running would take into consideration the running effects of the neutrino mixing parameters also appearing on the r.h.s. of these equations. The important parameter here is U_{e3} which could be very small at the weak scale and could attain a non-negligible value at the high scale. To trace the U_{e3} evolution, we can use the following effective operator:

$$m_\nu(\mu) = Y_\nu(\mu) M_R^{-1}(\mu) Y_\nu^T(\mu) \tag{2.8}$$

The RGEs for (2.8) are given in the literature [11, 12]. From them, it is possible to estimate the generated U_{e3} at high energy. For instance, in the case of hierarchical neutrino spectrum ($m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$) and barring the PMNS phases, one gets:[1]

$$\begin{aligned} \Delta U_{e3}^{hie}(M_W \to M_X) \approx & -\frac{1}{16\pi^2}\left[y_\tau^2 \ln\left(\frac{M_X}{M_W}\right) + y_t^2 \ln\left(\frac{M_X}{M_{R_3}}\right)\right] U_{e1}U_{e2}U_{\mu 3}U_{\tau 3}\frac{m_{\nu_2} - m_{\nu_1}}{m_{\nu_3}} \\ & \sim -(\tan^2\beta \cdot \mathcal{O}(10^{-6}) + \mathcal{O}(10^{-3})), \end{aligned} \tag{2.9}$$

where the first contribution $\propto y_\tau^2$ comes from the ordinary MSSM RG corrections, whereas the second one $\propto y_t^2$ is from neutrino Yukawa couplings above the seesaw scale. Thus even when $U_{e3} \ll 10^{-3}$ at the weak scale, there is a generated $\Delta U_{e3} \gtrsim 10^{-3}$ at the high scale, especially at the scale where it feeds into the slepton mass matrix. The RG-generated U_{e3} at the high scale would now become the dominant contribution to the LFV as long as the

[1] We will consider all parameters to be real and set phases to be zero in the present work. However there are some subtleties associated with such an assumption especially in the limit U_{e3} goes to zero, which we will elaborate in the text.

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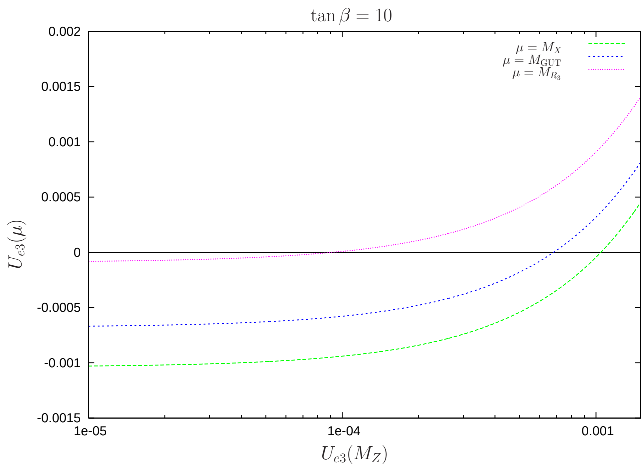

Figure 1: Behaviour of high-scale values of U_{e3} for small $U_{e3}(M_Z)$. The neutrino spectrum is hierarchical and $\tan\beta = 10$

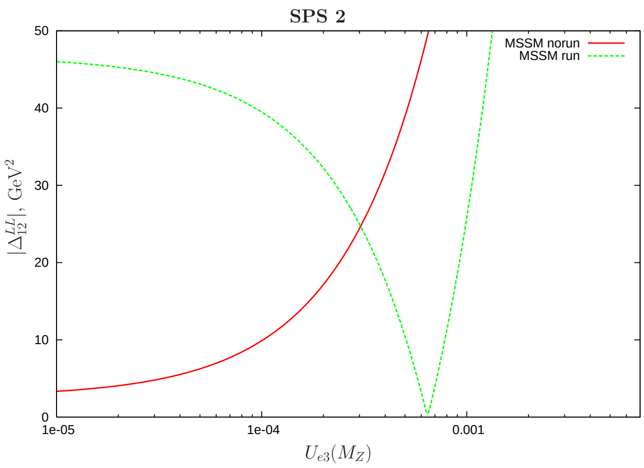

Figure 2: Behaviour of Δ_{12}^{LL} for small values of $U_{e3}(M_Z)$, in the point **SPS 2** ($m_0 = 1450\,$GeV, $M_{1/2} = 300\,$GeV, $A_0 = 0$, $\tan\beta = 10$) of the mSUGRA parameter space, with and without U_{e3} evolution.

high-scale value of U_{e3} overwhelms the limit value given by eq. (2.6). For $\tan\beta \sim 10$ we see that, even without the top-quark-like contribution, ΔU_{e3} far exceeds the limit value of U_{e3} (2.6). This RG generated contribution is *independent* of low-energy value of U_{e3} and would get generated at the high scale even when U_{e3} is zero. For larger values of U_{e3} this contribution would add to the low-energy number. This is best illustrated in the figure 1, where we plot the high-scale value of U_{e3} for a given value of U_{e3} at the weak scale[2] at three different high scales, M_{R_3}, $M_{\rm GUT}$ and M_X. As we see from the figure, U_{e3} at the high scale takes a constant value below $\sim \mathcal{O}(10^{-3} - \ 10^{-4})$ as a resultant of the RG correction as per eq. (2.9).

It is interesting to compare the contributions in eqs. (2.6) and (2.9). It is obvious that for reasonable values of tan β, even without the consideration of the contribution $\propto \ y_t^2$,

[2]For the rest of the neutrino parameters required for the running, we take $m_1 = 0.001\,$eV, $\Delta m^2_{\rm atm} > 0$.

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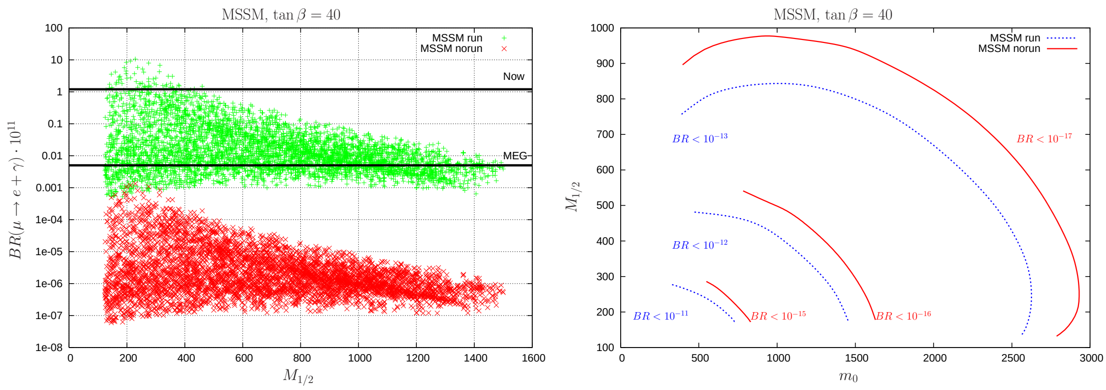

Figure 3: BR($\mu \to e + \gamma$) scatter and contour plot for $U_{e3}(M_Z) = 0$ ($\tan\beta = 40$, $\mu > 0$), as a comparison between the *running* and *non-running* U_{e3} cases. The scatter plot is made by scanning the mSUGRA parameters ($0 < m_0 < 5$TeV, $0 < M_{1/2} < 1.5$TeV, $-3m_0 < A_0 < 3m_0$) and keeping the points within an approximate LHC accessible region (i.e. $m_{\tilde{t}} \leq 2.5$TeV). For the contour plot $A_0 = 0$.

the RG generated U_{e3} is always larger than the limit value $U_{e3}^{\rm lim}$ irrespective of the value of the U_{e3} at the weak scale, even if it is zero. This implies that $(\Delta_{LL})_{12}$ will always have a constant contribution due to the RG generated U_{e3}. This is best demonstrated in figure 2 where we show the contrast in the variation of the $(\Delta_{LL})_{12}$ with respect to U_{e3} when neutrino running effects are taken into consideration or neglected. As we see from the figure, even with small or vanishing values of U_{e3} at the weak scale, $(\Delta_{LL})_{12}$ has a constant value which is much larger than the value of $(\Delta_{LL})_{12}$ without taking into consideration the running effects.

The evaluation of the contribution related to the running of U_{e3} involves a subtlety which manifests itself in the above figure. This subtlety arises because of the evolution of U_{e3} and the unknown CKM-like phase, δ of the $U_{\rm PMNS}$ matrix.[3] Note that in the limit U_{e3} goes to zero, δ remains undefined. And the RGE for the δ diverges [11]. In the present work, as we scan U_{e3} for very small values starting from zero, U_{e3} takes values which are negative at the high scale. Given that we have set all the phases to be zero in our work, this would correspond to the phase δ assuming a value π at the high scale, if we insist on the standard CKM-like parameterisation for the $U_{\rm PMNS}$ matrix to be valid also at the high scale, where all the angles are defined to be in the first quadrant ($0 < \theta_{13} < \pi/2$). Thus δ jumps from zero to π after the inclusion of RGE corrections.[4] Hence the contributions to $(\Delta_{LL})_{12}$ proportional to y_t^2 and y_c^2 in eq. (2.5) have opposite signs. As a consequence cancellations occur between the two contributions at the high scale as U_{e3} is varied. The exact cancellation occurs when the high scale U_{e3} value takes the $U_{e3}^{\rm lim}$ (2.6). A dip occurs in the $(\Delta_{LL})_{12}$ when this happens as seen in figure 2 and correspondingly in the branching ratio.[5]

[3]In standard notation $U_{e3} \equiv \sin\theta_{13} e^{i\delta}$. Here we use U_{e3} and $\sin\theta_{13}$ interchangeably, since we set the phases to zero.

[4]For a nice discussion of this point, see [11].

[5]For the impact of Majorana phases in BR($\mu \to e + \gamma$), please see [13].

Finally, we demonstrate the effect of taking into account the RG corrections to the neutrino mass matrix on the scatter/exclusion plots of the MSSM with right-handed neutrinos (RNMSSM) parameter space in figures 3. We can see from the scatter plot that branching ratio increases by at least three orders of magnitude once running effects are taken into account, which can be traced to ΔU_{e3} which is an order or two larger than the $U_{e3}^{\rm lim}$. For large tan β this increase is very significant: it brings most of the parameter space into the realm of MEG experiment (this is apparent in the scatter plot of figure 3 where $\tan\beta = 40$). In terms of the exclusion plots in the $(m_0, M_{1/2})$ plane, we see that taking into consideration the running effects largely enhances the region of parameter space probed indirectly by the MEG experiment. While the above two plots exhibit the particularly sizable effect of including the running of U_{e3} when $\tan\beta$ is large, it should be noted that these effects are always present and would be significant for whatever value of $\tan\beta$.

Finally before we close this section, a few comments are in order. In the above analysis, we have parameterized the unknown U_{e3} by considering it as a free input parameter. We have further considered the limit where it tends to zero at the low scale to illustrate the effect of small values of U_{e3} on the $\mathrm{Br}(\mu \to e, \gamma)$. This has been done with a purely phenomenological perspective, without dealing with the possible flavour symmetries giving rise to the present experimentally determined form of the $U_{\rm PMNS}$ matrix.

The limit $U_{e3}(M_Z) \to 0$ which we had discussed in this section, however needs further clarifications in this respect.[6] In the limit $U_{e3} \to 0$, the neutrino mass matrix can be thought of as having an additional (possibly discrete) symmetry [14].[7] Unless this symmetry is broken, a non-zero value for U_{e3} cannot be achieved. Let's note that the simplest symmetries like the $\mu \leftrightarrow \tau$ [15] are not really compatible with the SO(10) seesaw framework and Yukawa identification we have considered here.[8] In case there exists a flavour symmetry which does lead to $U_{e3}(M_X) = 0$, then one can assume that there exists some flavon fields breaking this symmetry, leading to a nonzero value of U_{e3} already at high scale. Below such breaking scale the flavour symmetry is no more effective and the RGEs are exactly as given in [11, 12]. If the flavour symmetry breaking also has a radiative origin, then effects could be similar in magnitude with RG effects. Thus, one can imagine partial cancellations between these two effects leading to small values of U_{e3} at the weak scale, without requiring a large fine tuning (as in the case $U_{e3}(M_Z) \sim 10^{-4}$). On the other hand, U_{e3} itself can also be purely of radiative origin. Finally we note here what we need is only small value of U_{e3}: $0 < U_{e3}(M_Z) < 10^{-3}$, as in our case U_{e3} decreases from M_X to M_Z (due to absence of phases, the RG running carries the same sign at all stages), with the limit value $U_{e3} \to 0$ only used to emphasize this case.

3. Running U_{e3} and double flavour violating MI in GUTs

So far most of the discussion in the previous sections has been focused on the impact of

[6]Incidentally, let's note that the current best fit value is $U_{e3} = 0$ [9].

[7]This statement is typically defined in the basis where charged lepton mass matrix is diagonal.

[8]The required $\mu \leftrightarrow \tau$ symmetric structure of the Y_ν matrix cannot, for instance, be satisfied in the case of the Y_ν eigenvalues having the same hierarchy of the Y_u.

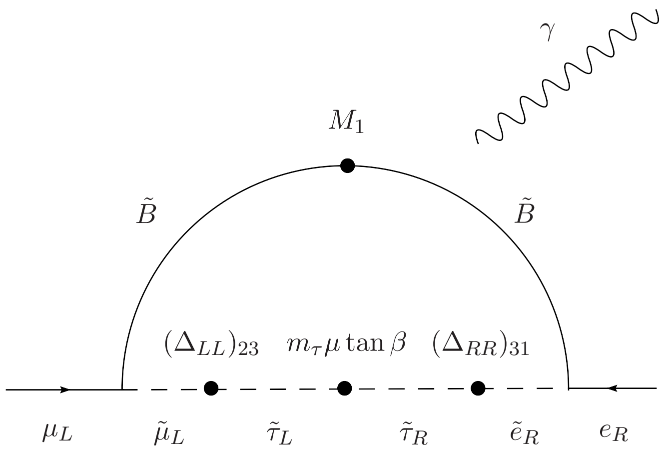

Figure 4: Feynman diagram contributing to the double MI of eq. (3.1).

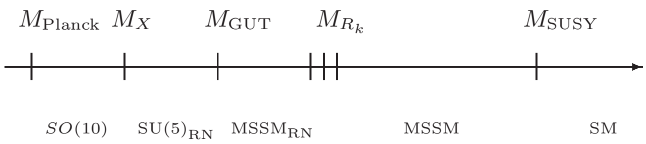

Figure 5: Schematic picture of the energy scales involved in the model.

taking into consideration the RG effects of neutrino mass matrix within the context of RNMSSM. There we have assumed a U_{e3}^{lim} value which is similar in the context of SUSY-GUTs. However in SUSY-GUTs, additional RG running effects between M_X and M_{GUT} exist which can become dominant in some regions of the parameter space.

In terms of the so-called mass-insertion approximation [16], the main such contribution is a double mass-insertion which generates an 'effective' LR flavour violating mass entry [5]:

$$(\delta_{LR})_{21}^{\text{eff}} = (\delta_{LL})_{23} \cdot \mu m_\tau \tan\beta \cdot (\delta_{RR})_{31} \tag{3.1}$$

where the $\delta_{ij} \equiv \Delta_{ij}/m_{\tilde{l}}^2$; where Δ_{ij} is already defined in eq. (2.4) of the slepton mass matrix and $m_{\tilde{l}}^2$ is the average slepton mass. The origin of such double mass-insertion is best depicted in the Feynman diagram in figure 4 [17]. This contribution, which is independent of U_{e3} would provide a flat contribution to the branching ratio irrespective of the value of U_{e3} at the weak scale. To discuss in detail, we will work in the SO(10) framework, where SO(10) is broken down to the Standard Model through an intermediate scale of SU(5) located at around 10^{16} GeV. The various scales involved here can be summarised in the figure 5.

Given the impact of the U_{e3} running in SUSY-seesaw (section II), one would expect that the U_{e3} proportional contribution would be the dominant force within the SUSY-GUT framework as the neutrino mass matrix running effects are larger. However, the double flavour violating MI eq. (3.1) which is independent of U_{e3} could become dominant in some regions of the parameter space. In these regions the running of U_{e3} would have no

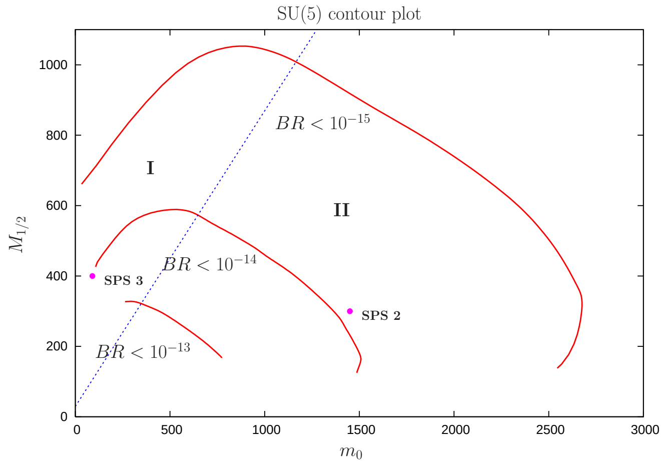

Figure 6: $U_{e3}(M_Z) = 0$ contour plots at fixed $\text{BR}(\mu \to e + \gamma)$ with $A_0 = 0$, $\tan\beta = 10$ for SU(5) ($\Delta m_{\text{atm}} > 0$, $m_1 = 10^{-3}$eV). Two regions of the $(m_0, M_{1/2})$ plane are defined: I where the dominant contribution to $\text{BR}(\mu \to e + \gamma)$ is given by $(\delta_{LL})_{23}(\delta_{RR})_{13}$; II where $(\delta_{LL})_{12}$ dominates.

strong impact on the total branching ratio. The interplay between these two effects is best demonstrated in figure 6.[9]

Figure 6 shows the contour plots at fixed $\text{BR}(\mu \to e+\gamma)$ in the plane $(m_0, M_{1/2})$ of the mSUGRA parameter space, for $U_{e3} = 0$, normal neutrino hierarchy and lightest neutrino mass $m_1 = 10^{-3}$eV; the other mSUGRA parameters are set to be: $A_0 = 0$, $|\mu| > 0$ and $\tan\beta = 10$. The diagonal line in the center of the figure are the points where the contribution of the double insertion $(\Delta_{LL})_{23}(\Delta_{RR})_{13}$ is equal to the $(\Delta_{LL})_{12}$. This line divides into two regions the $(m_0, M_{1/2})$ plane: region I where the double insertion dominates, and region II where $(\Delta_{LL})_{12}$ forms the main contribution. Thus, though generically, $(\Delta_{LL})_{12}$ dominates the amplitudes for $\mu \to e + \gamma$, for extremely small values of U_{e3}, the contributions of $(\Delta_{LL})_{12}$, enhanced by the running of U_{e3}, and the double MI can be competing in some regions of the SUSY parameter space.

The benchmark points **SPS 3** ($m_0 = 90\,\text{GeV}$, $M_{1/2} = 400\,\text{GeV}$, $A_0 = 0$, $\tan\beta = 10$) and **SPS 2** ($m_0 = 1450\,\text{GeV}$, $M_{1/2} = 300\,\text{GeV}$, $A_0 = 0$, $\tan\beta = 10$) lie respectively in region I and in region II. The competition between these two contributions is evident in figure 7, where $\text{BR}(\mu \to e + \gamma)$ is plotted as a function of $U_{e3}(M_Z)$ considering only one contribution at a time (i.e. putting the other ones to zero) in the two different regions I (**SPS 3**) and II (**SPS 2**). As we can see from the figures, in both the cases, above $\sim\ 10^{(-3)}$, the $(\Delta_{LL})_{12}$ contribution dominates. However, below that value, in the case of **SPS 2**, the running effects of U_{e3} become very crucial, whereas in the case of **SPS 3**, the double flavour violating mass insertion dominates. Finally we note that the U_{e3} dependent minima in the $(\Delta_{LL})_{12}$ contribution are due to the cancellation of two dominant contributions as

[9] In making this figure, we have taken, $M_{\text{GUT}} \sim 2 \cdot 10^{16}$GeV, $M_X \sim 5 \cdot 10^{17}$GeV. The numerical routine computes the rates of LFV processes by using the exact masses and mixings of the SUSY particles, obtained from full 1-loop RGE evolution of the mSUGRA parameters. The high-energy values of fermion masses and mixings are set by evolving them from the e.w. scale up to M_X. For more details about the numerical routine, we refer to [5].

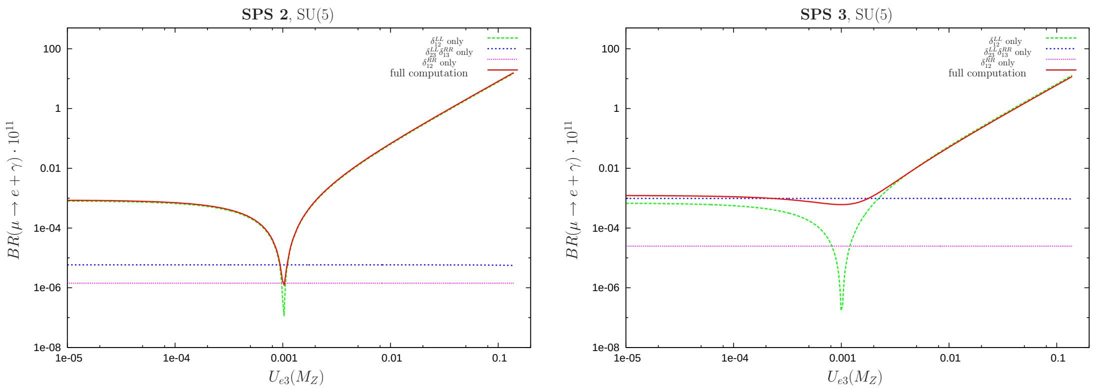

Figure 7: Different SU(5) contributions to BR($\mu \to e+\gamma$) for the benchmark points, as a function of low-energy U_{e3}.

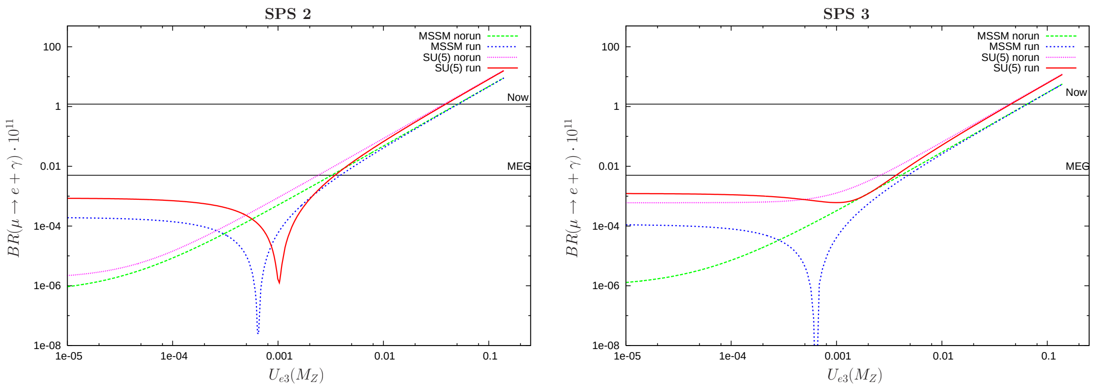

Figure 8: BR($\mu \to e+\gamma$) as a function of $U_{e3}(M_Z)$ in the **SPS 2**, **SPS 3** benchmark points of the mSUGRA parameter space ($\Delta m_{\rm atm} > 0$, $m_1 = 10^{-3}$eV). The plots show the effect of switching on the U_{e3} running both for MSSM and SU(5).

discussed in the previous section.

It would be interesting to compare the effect of running in both the case of GUT theories and in RNMSSM. In figure 8, we plot the total branching ratios with and without taking the running effects for case of RNMSSM and SUSY-SU(5). In RNMSSM the U_{e3} running gives, for small low-energy values, an order of magnitude enhancement of the BR with respect to the sub-dominant y_c contribution. In SU(5) such enhancement is almost hidden by pure SU(5) effect in the case of **SPS 3** that lies in region I, while it results dominant for **SPS 2**.

So far we have been discussing the RG effects of U_{e3} within the context of hierarchical neutrino spectrum. Different light neutrinos spectra could consistently change the above results. While in the case of degenerate spectrum (lightest neutrino mass $\gtrsim$ 0.1eV), we found similar results to the normal hierarchy (even if the degenerate case should be more sensitive to the change of phases). The so-called inverted hierarchy ($\Delta m_{\rm atm} < 0$, $m_{\nu_3} = 10^{-3}$eV) doesn't give enhancement effects due to the U_{e3} running comparable to the normal hierarchy case. This is as expected from the direct proportionality of ΔU_{e3} to the lightest neutrino mass m_{ν_3} [11]. Moreover in this latter case, the scales of right-handed neutrinos

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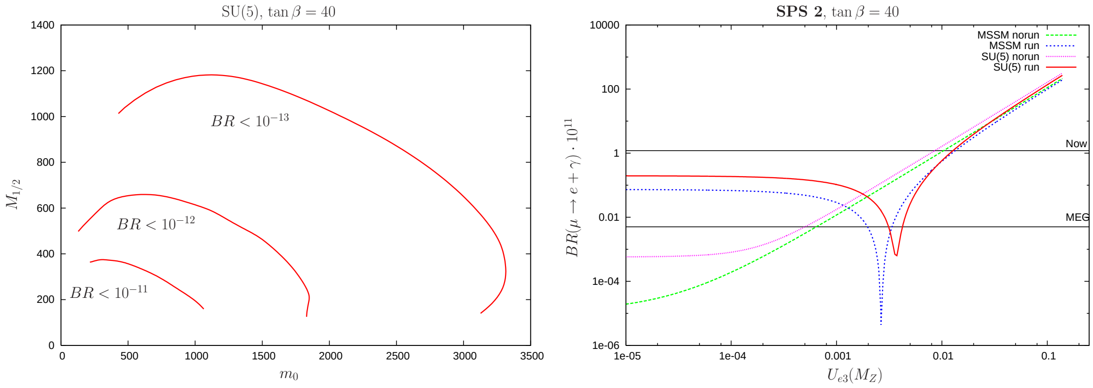

Figure 9: $U_{e3}(M_Z) = 0$ contour plot and $\mathrm{BR}(\mu \to e + \gamma)$ vs. $U_{e3}(M_Z)$ for $\tan\beta = 40$. The point considered for the second plot is **SPS 2** with $\tan\beta = 40$ instead of 10.

are much closer to the GUT scale and thus even the pure SU(5) effects coming from double MI which can enhance the BR over the y_c contribution are smaller compared to the normal hierarchical case.

Finally, we consider what happens in the case of $\tan\beta = 40$. We find that for small values of U_{e3} the dependence of the branching ratio on $\tan\beta$, where $(\delta_{LL})_{12}$ gives the dominant contribution, is not the usual $\propto (\tan\beta)^2$, because $\tan\beta$ would also affect the running of U_{e3} (and $(\delta_{LL})_{12} \propto U_{e3}$). The result is that the enhancement of branching ratio at $\tan\beta = 40$ with respect to $\tan\beta = 10$ is much larger than the usual scaling factor of 16. This can be seen in figure 9, where the SU(5) contour plot and $\mathrm{BR}(\mu \to e + \gamma)$ for **SPS 2** with $\tan\beta = 40$ instead of 10 are plotted.

4. Final remarks

The unknown neutrino mixing angle U_{e3} is an object of much speculation and interest for neutrino mass model builders as well as for experimentalists. While we have no clue on the value of this angle, except for an upper bound of $|U_{e3}| \lesssim 0.14$, it could as well take very small values even reaching zero at the weak scale. Over the past few years, various strategies have been devised to probe U_{e3} down to values of $\mathcal{O}(10^{-2})$ [18]. While these experiments probe U_{e3} at weak scale values, SUSY-seesaw based models give information about the high-scale values of U_{e3} through indirect measurements of decay rates such as $\mu \to e + \gamma$ at dedicated facilities like MEG [19]. In the present paper, we have stressed the importance of considering RG running effects on the neutrino mass matrices while making such a correlation between weak scale measurements and high-scale probes of U_{e3}, which has been neglected in earlier works [20].

Let's make a final consideration on the link between the value of U_{e3} (at the low-energy scale at which we hope to measure it soon) and $\mathrm{BR}(\mu \to e + \gamma)$. The key-point is the assumption that the angles entering the diagonalization of the neutrino Yukawa matrix Y_ν are linked to those connected with the diagonalization of the mass matrix of physical neutrinos, i.e. the PMNS angles. If this is the case, then, independently of what we assume for the value of the Y_ν eigenvalues, it is possible that the running of U_{e3} from the electroweak scale up to M_X induces an effect on it of $\mathcal{O}(\tan^2\beta \cdot 10^{-6})$. Hence, even if future measurements of U_{e3} would lead to very small values of it, such running effects could provide contributions to U_{e3} so large that a $\mathrm{BR}(\mu \to e + \gamma)$ accessible to the MEG experiment could occur if some large neutrino coupling is present. This is the main point of our present analysis where a top-down approach with an underlying SO(10) symmetry is taken. An analogous investigation [7] where a bottom-up phenomenological approach was considered also had similar conclusions. The main difference between the two approaches concerns the information about the size of the neutrino Yukawa couplings. In our SO(10) framework, we can correlate Y_ν with Y_u, hence obtaining the large contribution from the running from M_R up to M_X in eq. (2.5). Also, assuming such a GUT underlying structure allowed us to include the effects due to the running above M_{GUT} of eq. (3.1).

In conclusion, let us stress again that taking running effects into account could in principle lead to a "constant" enhancement of the value of U_{e3} at the high scale, bringing $\mu \to e + \gamma$ into the realm of MEG for SUSY parameter space regions which were previously excluded without the inclusion of such running.

We have not addressed the important and interesting issue of origins of neutrino mixing angles particularly U_{e3}, treating it as a free parameter. The question of low values of U_{e3} at the weak scale, when the radiative corrections themselves are as large as $\sim 10^{-3}$ would deserve more attention as it points out to cancellations within the parameters of the model and radiative corrections. We hope to deal with this issue at a later date.

The correlation of U_{e3} and flavour violating effects continue to be important in the context of SUSY-GUTs and any measurement of flavour violation at MEG could lead to shedding some light on either U_{e3} or on the parameter space of SUSY-GUTs.

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